E7.3 Material index for a light, strong beam (Figure E5). In stiffness-limited applications, it is elastic deflection that is the active constraint: it limits performance. In strength-limited applications, deflection is acceptable provided the component does not fail; strength is the active constraint. Derive the material index for selecting materials for a beam of length *L*, specified strength and minimum weight. For simplicity, assume the beam to have a solid square cross-section *t x t.* You will need the equation for the failure load of a beam (Appendix B, Section B4). It is

$$F_f = \frac{I\sigma_f}{y_m L}$$

where y_m is the distance between the neutral axis of the beam

and its outer filament and $I = t^4/12 = A^2/12$ is the second moment of the cross-section. The table itemizes the design requirements.

Figure E5

Function	• Beam
Constraints	• Length L is specified
	• Beam must support a bending load F without yield or fracture
Objective	• Minimize the mass of the beam
Free variables	• Cross-section area, A
	• Choice of material